# Section 1.5: Exponential and Logarithmic Functions

In this section, we examine exponential and logarithmic functions. We use the properties of these functions to solve equations involving exponential or logarithmic terms, and we study the meaning and importance of the number .

## Exponential Functions

In general, any function of the form , where , , is an **exponential function** with base and exponent . Exponential functions have constant bases and variable exponents

Note that a function of the form for some constant is not an exponential function but a power function.

### Evaluating Exponential Functions

**Law of Exponents**

For any constants , , and for all and .

Note: these are only some of the properties. Make sure you know the others as well!

### Graphing Exponential Functions

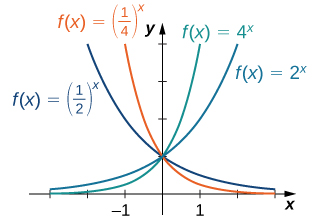
For any base , , the exponential function is defined for all real numbers and . The following characteristics hold true:

Domain:

Range:

If , then is increasing on

If , then is decreasing on



Examples

1. Suppose a particular population of bacteria is known to double in size every 4 hours. If a culture starts with 1000 bacteria, the number of bacteria after 4 hours is . The number of bacteria after 8 hours is . In general, the number of bacteria after hours is . Letting , we see that the number of bacteria after hours is . Find the number of bacteria after 6 hours, 10 hours, and 24 hours.
2. Use the laws of exponents to simplify each of the following expressions:
3. For the function, , sketch the graph and then determine the domain, range, and horizontal asymptote.

## The Number

To six places of accuracy,

Functions involving base arise often in applications, we call the function the **natural exponential function**. Other ways you have seen also pertains to applications involving money. For example, if a person puts dollars in an account at an annual interest rate , compounded continuously, then .

Example

Suppose in invested in an account at an annual interest rate of , compounded continuously.

1. Let denote the number of years after the initial investment and denote the amount of money in the account at time . Find a formula for .
2. Find the amount of money in the account after 10 years and after 20 years.

## Logarithmic Functions

Using our understanding of exponential functions, we can discuss their inverses, which are the logarithmic functions. These come in handy when we need to consider any phenomenon that varies over a wide range of values, such as pH in chemistry or decibels in sound levels.

The exponential function is one-to-one, with domain and range . Therefore, it has an inverse function, called the **logarithmic function** with base b. For any , , the logarithmic function with base , denoted , has domain and range , and satisfies

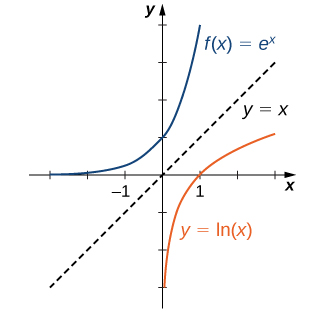
if and only if .

The most commonly used logarithmic function is the function . Since this function uses natural as its base, it is called the **natural logarithm**. Here we use the notation or to mean .

Since the functions and are inverses of each other,

and ,

and their graphs are symmetric about the line .



Before solving some equations involving exponential and logarithmic functions, let’s review the basic properties of logarithms.

**Properties of Logarithms**

If , , and is any real number, then

1. (**Product property**)
2. (**Quotient property**)
3. (**Power property**)

Examples

1. For each of the following, write the equation in equivalent exponential/logarithmic form.
2. For the function, , sketch the graph and then determine the domain, range, and vertical asymptote.
3. For each of the following, use the properties of logarithms to write the expressions as a sum, difference, and/or product of logarithms.
4. Solve each of the following equations for .
5. Solve each of the following equations for .

When evaluating a logarithmic functions with a calculator, you may have noticed that the only options are , or log, called the common logarithm, or , which is the natural logarithm. However, exponential functions and logarithm functions can be expressed in terms of any desired base . If using a calculator, you may need to use the change of base formulas.

**Change-Of-Base Formulas**

Let , and .

1. for any real number .

If , this equation reduces to .

1. for any real number .

If , this equation reduces to .

Example: For each of the following, use the change-of-base formula and either base 10 or base to evaluate the given expressions. Answer in exact form and in approximate form, round to four decimal places.